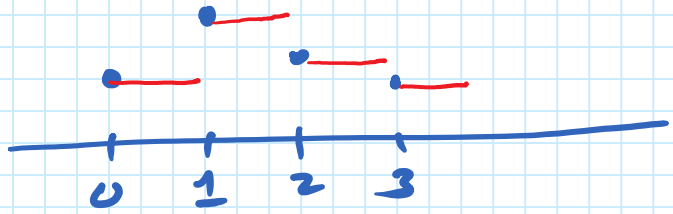
Interpolation (textbook Chapter 9)

Demo: Adobe Illustrator Pen tool

November 6, 2017 10:14 AM

§ Constant Interpolation

Everything in 1D



eg. Toon Shading

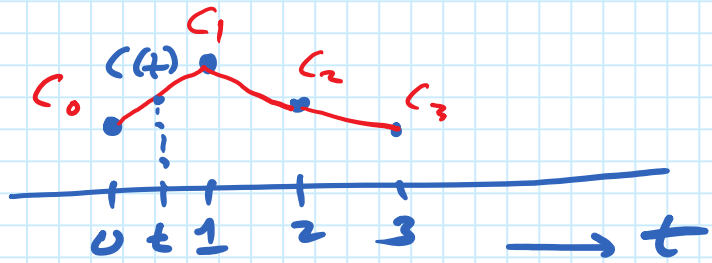
Pixels in an image

Positive: Easy

Negative: Discontinuous

Cause visual artifacts

§ Linear Interpolation



Problem of Interpolation: find $C(t)$ given discrete values $C_0, C_1, \ldots$

$$C(t) = at + b$$

Parametric equation of a straight line

+ Boundary conditions

$$C(0) = C_0 = 0 + b \quad \text{so} \quad b = C_0$$

$$C(1) = C_1 = a \cdot 1 + b \qquad a = C_1 - C_0$$

So $\boxed{C(t) = (C_1 - C_0)\,t + C_0}$ (1)

November 6, 2017 10:28 AM

Rewrite Eqn ① as

Data

Blending functions (a kind of basis)

$$\boxed{C(t) = C_0 \underbrace{(1-t)}_{b_0(t)} + C_1 \underbrace{t}_{b_1(t)}}$$

Looks trivial, but key step ☆☆

Generalizes easily to higher dimensional data and to "higher order" interpolation

↳ refers to the order to the polynomial in t

§ Higher dimension.

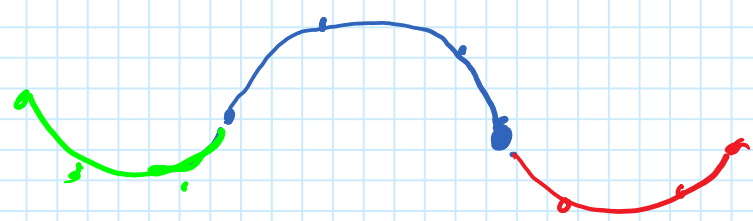
Trivial. Just use higher dimensional array for data

$$C_i = \begin{bmatrix} \\ \\ \\ \end{bmatrix} \begin{matrix} 1 \\ \vdots \\ \vdots \\ n \end{matrix}$$

§ Higher degree polynomials

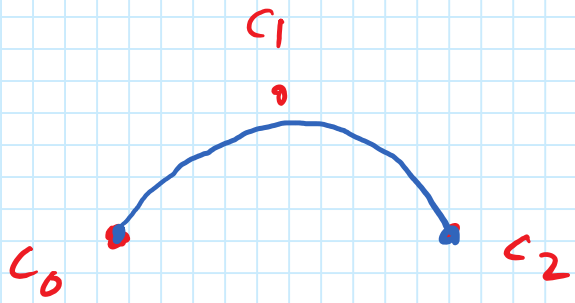
	quadratic	cubic	quartic	quintic	..
deg =	2	3	4	5	...

Sweet spot



Can connect a sequence of curves together to create "Splines" (called "paths" in Illustrator)

Quadratic: $c(t) = c_0\, b_0(t) + c_1\, b_1(t) + c_2\, b_2(t)$



polynomials

$(1-t)^2 \quad 2(1-t)t \quad t^2$

This a <u>Bézier curve</u>

Cubic:

b_0	b_1	b_2	b_3
$(1-t)^3$	$3(1-t)^2 t$	$3(1-t)t^2$	t^3

Next class: Shadows