CPSC 314
Computer Graphics

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Affine spaces:
frames, homogeneous coordinates,
translation

Announcements

- Quiz 1 on Friday (see last class notes for preparation tips)
- Office hour Sep 21 2-3pm X853
- Assignments
  - Lateness policy: up to **three days in the entire term** Submit well before your grading time slot
- Today:
  - Essential math for graphics
    (read Textbook Chapter 3)

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Quiz 1 Preparation

- CAREFULLY review lecture notes, and assignment 1
- Textbook. Read all of these, except as noted. But use class notation (see L4 and today for differences).
  - Ch 1
  - Ch 2: skip Section 2.5
  - Ch 3: skip Section 3.6

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Quiz Format

- The Quiz is closed book, closed electronic device (laptops, phones, etc. should be out of sight).
- 45 marks (in 45 minutes. Please be on time, will start at 10!)
- Three types of questions
  - **[12 marks]** small questions (fill in the blank, many choices given)
    "Can you recognize the concepts?"
  - **[23 marks]** hdirect questions (write down short answer)
    "Do you understand the concepts?"
  - **[10 marks]** problem solving questions
    "Can you use your knowledge in a new situation?"

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Quiz Format

- The first two question types are meant to be easy. Try to go through them quickly, so that you have time to think about the problem solving questions at the end.

- Some questions may have multiple parts that build on one another. Answer for part (a) is used in part (b), etc. Even if the answer for (a) is incorrect, you can get credit for later parts if you show your steps (i.e., later parts will be graded on the subsequent logic).

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- SWITCH TO TABLET

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Affine Spaces

(Recap last class)

- Homogeneous coordinates of a point $\begin{pmatrix} * \\ * \\ * \\ 1 \end{pmatrix}$

- Homogeneous coordinate of a vector

$$\text{as} \quad \vec{v} = \begin{pmatrix} \vec{b}_1 & \vec{b}_2 & \vec{b}_3 & \vec{b}_0 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} * \\ * \\ * \\ 0 \end{pmatrix}$$

Note: The key difference is whether the last element is zero or non-zero.

↙ vector

↘ point

By convention, the non-zero is normalized to 1.

Alternative view: Identify $\alpha \tilde{\vec{p}} = \tilde{\vec{p}}$
$\alpha \neq 0$

- Homogeneous coordinates of a <u>linear</u> transformation of 3D vectors?

$$\underline{L} = 4 \left[\begin{array}{c|c} \underline{\ell}_{3\times3} & \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \\ \hline 0 \quad 0 \quad 0 & 1 \end{array} \right]$$

⊛ Special structure

check $\vec{v}$ with coords $\begin{pmatrix} v_1 \\ v_0 \end{pmatrix}$ ⟿ $\underline{L}\,\tilde{\vec{v}} = \left(\underline{\ell} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \right)_3$

check $\vec{v}$ with coords $\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ $\rightsquigarrow$ $\underline{L}\,\bar{v} = \begin{pmatrix} \bar{\underline{1}} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \\ \overline{- \; 0 \; - - -} \end{pmatrix} \begin{matrix} 3 \\ \\ 1 \end{matrix}$

3×3 matrix as in L5

eg.
$$\underline{L}\begin{pmatrix} P_1 \\ r_2 \\ P_3 \\ 1 \end{pmatrix} = \left(\begin{array}{c} \overline{L}\begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix} \\ \hline 1 \end{array} \right)$$
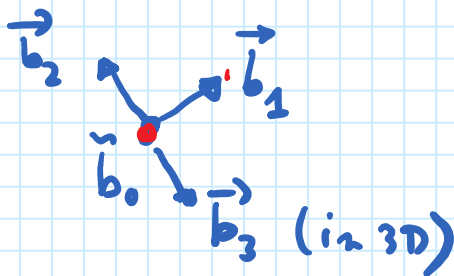
– Translating

$$3\begin{bmatrix} 1 & & & a_1 \\ & 1 & & a_2 \\ & & 1 & a_3 \\ \hline & & & 1 \end{bmatrix}\begin{pmatrix} P_1 \\ r_2 \\ P_3 \\ 1 \end{pmatrix} = \left(\begin{array}{c} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix} + \begin{array}{c} a_1 \\ a_2 \\ a_3 \end{array} \\ \hline 1 \end{array} \right)$$

$$= \overline{P} + \overline{a}$$

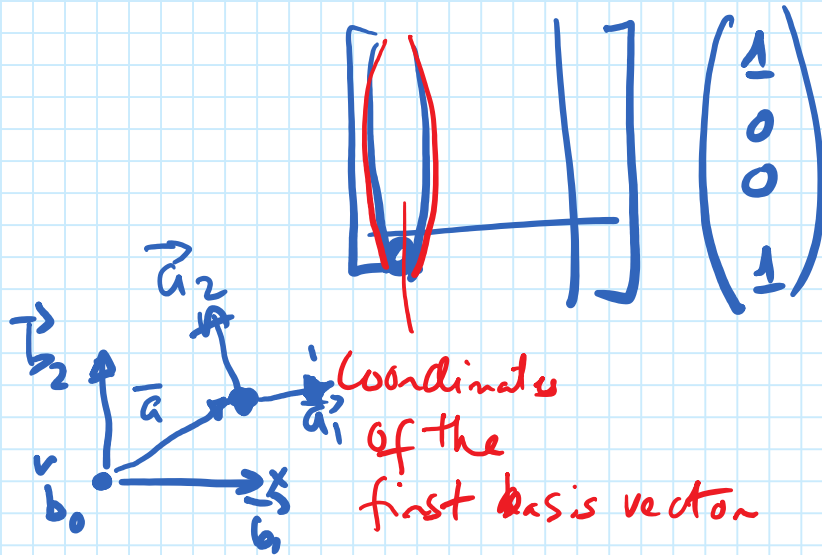$$\begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ 0 \end{pmatrix}$$

Frame

$$\widetilde{p} = \underline{\widetilde{b}}\, \overline{p}$$

$$\vec{a} = \underline{\widetilde{b}}\, \overline{a}$$

§ Visualizing frames

$\vec{b_3}$ (in 3D)

§ "Visualize" parts of $\underline{\underline{T}}$

$$\left[\; \Big| \; \Big| \; \right] \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Coordinates of the first basis vector

$$\left(\Big| \vec{a}_1 \Big| \; \Big| \vec{a}_2 \Big| \; \Big| \; \vec{} \; \Big| \; 1 \right) \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ 1 \end{pmatrix}$$

Any Homogeneous transformation matrix can visualized as a new frame