Two views of rotations.
Frame Transformations

Announcements

- Quiz 1 and Assignment 1 handback slightly delayed. Will be done by the end of the week
- Assignment 2 available later this week.
- Homework for today:
  - Read textbook Chapter 4, 5
Rotations, again

What is it about the matrix that makes it a rotation?

Each column and row has length 1

The columns and rows are orthogonal

Such a matrix is called "orthogonal" matrix

Terminology: "orthonormal" refers to basis \( \overrightarrow{b_1}, \overrightarrow{b_2}, \overrightarrow{b_3} \), "orthogonal" refers to a matrix.

Orthogonal matrices are nice!

\[
\overline{R}^T \overline{R} = \overline{I}
\]

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

So \( \overline{R}^{-1} = \overline{R}^T \)

Reflections are also orthogonal, but not rotations.

Rotations preserve "handedness".

\[
\det(\overline{R}) = +1 \quad \text{Rotation} \quad \text{only possible orthogonal}
\]
\[ \text{det} (R) = +1 \quad \text{Rotation} \]
\[ = -1 \quad \text{Reflection} \]

only possible orthogonal matrices
Frame Transformations

Two kinds of tasks

1. What are the coordinates of \( \mathbf{P} \) in frame \( \mathbf{\hat{a}} \)?
2. How do we move the model to another place?

How to define a new frame \( \mathbf{\hat{a}} \), relative to a known frame \( \mathbf{\hat{b}} \)?

The key is to realize that the physical point \( \mathbf{\tilde{p}} \) doesn’t change.

\[
\mathbf{\tilde{p}} = \mathbf{\hat{b}} \mathbf{\tilde{p}} = \mathbf{\hat{a}} \mathbf{\hat{a}}' \mathbf{\tilde{p}}' \]

given

\[
\begin{pmatrix}
\hat{a}_x \\
\hat{a}_y \\
\hat{a}_z \\
\hat{a}_0 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
\hat{b}_x \\
\hat{b}_y \\
\hat{b}_z \\
\hat{b}_0 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
\hat{a}_x & \hat{a}_y & \hat{a}_z & \hat{a}_0
\end{pmatrix}
\]

\[
\begin{pmatrix}
\tilde{b}_x \\
\tilde{b}_y \\
\tilde{b}_z \\
\tilde{b}_0 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
\tilde{a}_x & \tilde{a}_y & \tilde{a}_z & \tilde{a}_0
\end{pmatrix}
\]

\[
\begin{pmatrix}
\tilde{a}_x \\
\tilde{a}_y \\
\tilde{a}_z \\
\tilde{a}_0 \\
\end{pmatrix}
\]
\[- \left( \begin{array}{c} a_2 \\ a_1 \\ \vdots \\ a_0 \end{array} \right) \]
define as \[ \Delta \]
\[ \tilde{p} = \tilde{b} \, \tilde{\tilde{p}} = \tilde{\lambda} \, \tilde{p}' = \tilde{\lambda} \, \tilde{\tilde{A}} \, \tilde{p}' \]

Two views of what we just did, i.e.,

\[ \tilde{b} \, \tilde{\tilde{p}} = \tilde{\lambda} \, \tilde{\tilde{A}} \, \tilde{p}' \]

1. Defined a new frame \( \tilde{\tilde{A}} \)

2. We moved \( \tilde{p}' \) by \( \tilde{\tilde{A}} \) to sit \( \tilde{p} \)