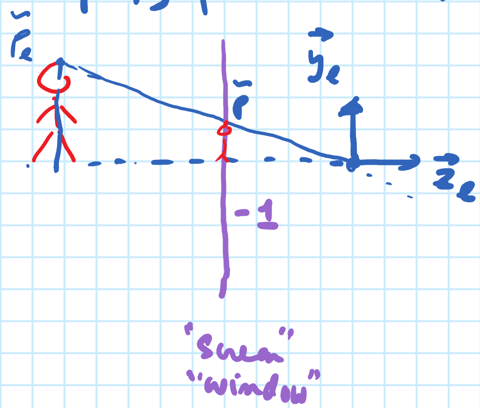
Cameras & Projection

October 6, 2017 9:57 AM

The textbook discussion is different from mine

To simplify presentation, mostly look at 2D

$$\bar{p}_e = \begin{pmatrix} y \\ z \\ 1 \end{pmatrix}$$



x will behave similar to y

$$\bar{p} = \begin{pmatrix} y_p \\ z_p \\ 1 \end{pmatrix} = ?$$

$z_p = -1$ by design

$y_p = -\frac{y}{z}$ because $\frac{y_p}{y} = \frac{-1}{z}$ (similar triangles)

This is a non-linear transform!

Good News: homogeneous coordinates can handle this

—— x ——

§ Orthographic Projection

- simpler than the perspective / central projection
- We'll convert central projection into this.
- Useful in applications in Architecture and Engineering

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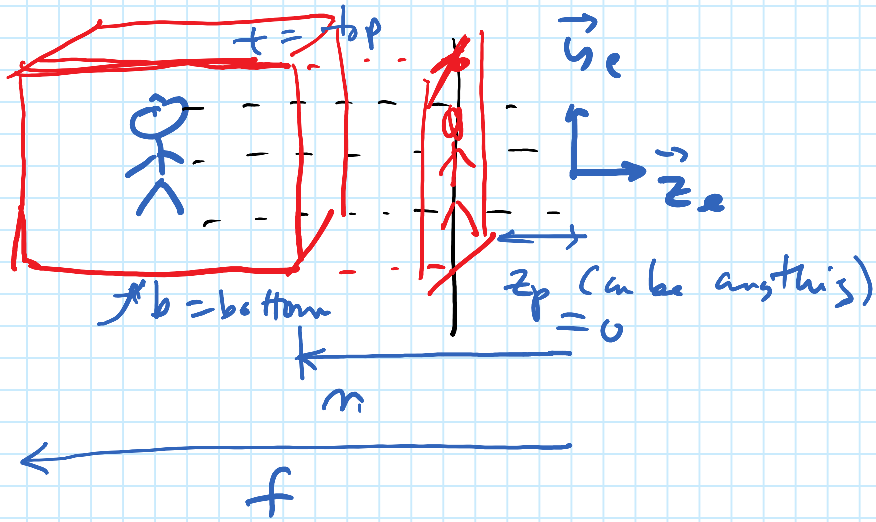


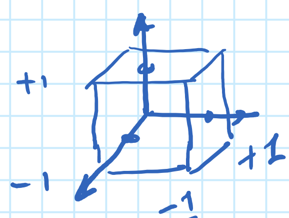
Project parallel to $\vec{z}_e$

What does the projection matrix look like?

$$\begin{pmatrix} y \\ z \\ 1 \end{pmatrix} \xrightarrow{N} \begin{pmatrix} y \\ 0 \\ 1 \end{pmatrix} \qquad N = \left(\begin{array}{cc|c} 1 & & \\ & 0 & \\ \hline & & 1 \end{array}\right)$$

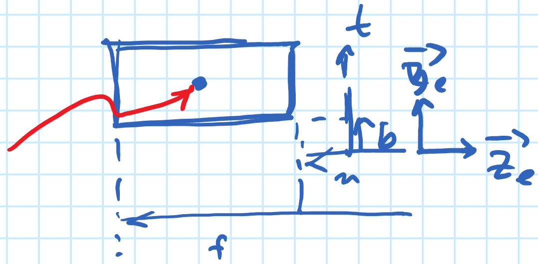
To view the "View frustum" (box in this case) we want to transform it eventually to fit on the screen.

So we transform the box into "<u>Normalized Device Coordinates</u>"



<u>Step 1</u>: Translate the "View Box" to origin

$$\begin{pmatrix} \frac{t+b}{2} \\ \frac{n+f}{2} \end{pmatrix}$$

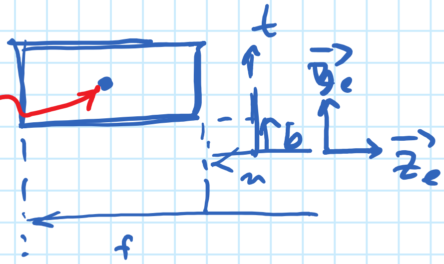


Move to origin using

$$T = \left[\begin{array}{cc|c} 1 & & -\frac{t+b}{2} \\ & 1 & -\frac{n+f}{2} \\ \hline & & 1 \end{array}\right]$$

Step 1: Translate the "View^{-1} Box" to origin

$$\begin{pmatrix} \frac{t+b}{2} \\ \frac{n+f}{2} \end{pmatrix}$$



Move to origin using

$$T = \left[\begin{array}{cc|c} 1 & & -\frac{t+b}{2} \\ & 1 & -\frac{n+f}{2} \\ \hline & & 1 \end{array}\right]$$

Step 2: Scale to make the half-size $= 1$

e.g. along y box is $\frac{t-b}{2} \rightarrow 1$

$$S = \left[\begin{array}{cc|c} \frac{2}{t-b} & & \\ & \frac{2}{n-f} & \\ \hline & & 1 \end{array}\right]$$

Projection matrix

$$P_o = ST = \left[\begin{array}{cc|c} \frac{2}{t-b} & & -\frac{t+b}{t-b} \\ & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ \hline & & 1 \end{array}\right]$$

Exercise: do this for the x-coordinate

Perspective

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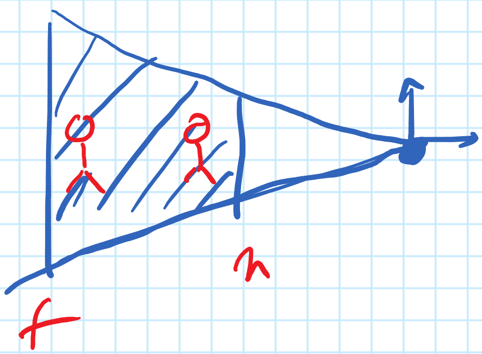

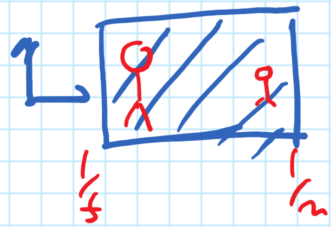

Projective Transformation
("unhinging" transform)

Depth information is retained,
even though distorted