Projector Textures, Sampling 1

Dinesh K. Pai

Textbook Chapters 15.4, 16

Several slides courtesy of M. Kim

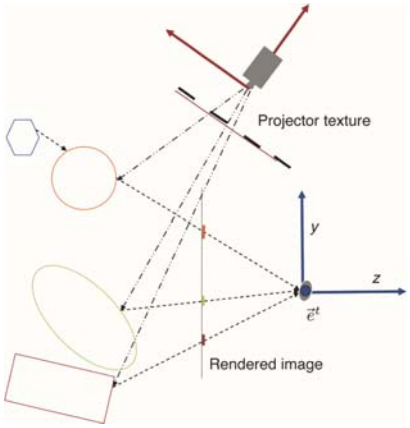
Today

- Announcements
  - No class on Monday. Remembrance Day
  - Reminder: Quiz 3 on Friday Nov 17
  - I will post a couple of Quiz 3 practice questions on Piazza by Tuesday. Will discuss answers on Wednesday
- Projector Texture mapping
- Sampling and Aliasing

Projector texture mapping

- There are times when we wish to glue our texture onto our triangles using a *projector* model, instead of the affine gluing model.
- For example, we may wish to simulate a slide projector illuminating some triangles in space.

Geometry of Projector Textures

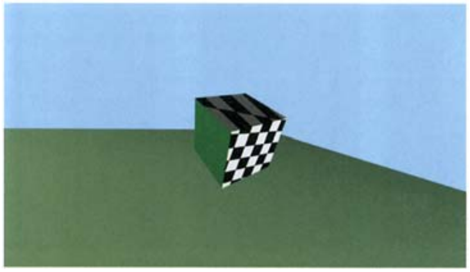


Transformations are similar to shadow mapping

Projector texture mapping

- The slide projector is modeled using 4 by 4, modelview and projection matrices, M_s and P_s

$$\begin{bmatrix} x_t w_t \\ y_t w_t \\ - \\ w_t \end{bmatrix} = P_s M_s \begin{bmatrix} x_o \\ y_o \\ z_o \\ 1 \end{bmatrix}$$

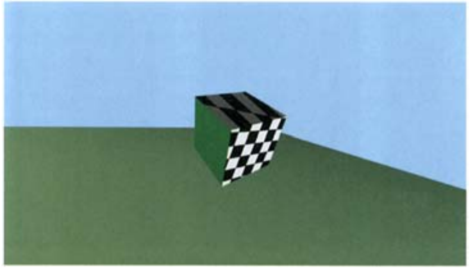


Projector texture mapping

- With the texture coordinates defined as

$$x_t = \frac{x_t w_t}{w_t} \text{ and } y_t = \frac{y_t w_t}{w_t}$$

- To color a point on a triangle with object coordinates $[x_o, y_o, z_o, 1]^t$, we fetch the texture data stored at location $[x_t, y_t]^t$



Projector texture mapping

- The three quantities $x_t w_t$, $y_t w_t$ and w_t are all affine functions of (x_o, y_o, z_o). Thus these quantities will be properly interpolated over a triangle when implemented as varying variables.
- In the fragment shader, we need to divide by w_t to obtain the actual texture coordinates.
- When doing projector texture mapping, we do not need to pass any texture coordinates as attribute variables to our vertex shader.

Projector texture mapping

- We simply use the object coordinates already available to us, and compute the texture coordinates.
- We do need to pass in, using uniform variables, the necessary projector matrices.

Projector texture mapping

- Projector vertex shader

```
#version 330

uniform mat4 uModelViewMatrix;
uniform mat4 uProjMatrix;

uniform mat4 uSProjMatrix;
uniform mat4 uSModelViewMatrix;

in vec4 aVertex;
out vec4 vTexCoord;

void main(){
  vTexCoord = uSProjMatrix * uSModelViewMatrix * aVertex;
  gl_Position = uProjMatrix * uModelViewMatrix * aVertex;
}
```

Vertex shader generates texture coordinates! But not normalized

Projector texture mapping

- Projector fragment shader

```
#version 330

uniform sampler2D vTexUnit0;

in vec4 aTexCoord;
out vec4 fragColor;

void main(){
  vec2 tex2;
  tex2.x = vTexCoord.x/vTexCoord.w;
  tex2.y = vTexCoord.y/vTexCoord.w;
  vec4 texColor0 = texture2D(vTexUnit0, tex2);
  fragCoor = texColor0;
}
```

Projector texture mapping

- Conveniently, OpenGL even gives us a special call texture2DProj(vTexUnit0, pTexCoord), that actually does the divide for us.
- Inconveniently, when designing our slide projector matrix uSProjMatrix, we have to deal with the fact that the canonical texture image domain in OpenGL is the unit square, whose lower left and upper right corners have coordinates $[0,0]^t$ and $[1,1]^t$ used for the display window.

Texture mapping tips and learning resources

- Read Texture Viewport (Textbook 12.3)
- Check out this excellent demo of transformations: http://www.realtimerendering.com/udacity/transforms.html
- Nice online animations of many things we cover in this course, esp. related to textures http://acko.net/files/fullfrontal/fullfrontal/webglmath/online.html

Sampling

Two views of images

- A *continuous image,* $I(x_w, y_w)$, is a bivariate function.
  - range is a linear color space.
- A *discrete image* I[i][j] is a two dimensional array of color values.
- We associate each pair of integers i, j, with the continuous image coordinates $x_w = i$ and $y_w = j$

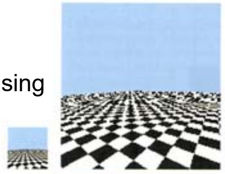
Sampling

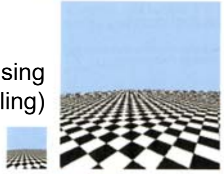
- The simplest and most obvious method to go from a continuous to a discrete image is by *point sampling.*
- To obtain the value of a pixel i, j, we sample the continuous image function at a single integer valued domain location:

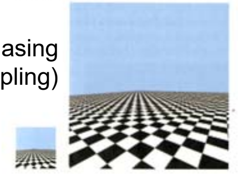
$$\mathrm{I[i][j]} \leftarrow I(i,j)$$

- This can results in unwanted artifacts.

Aliasing and anti-aliasing

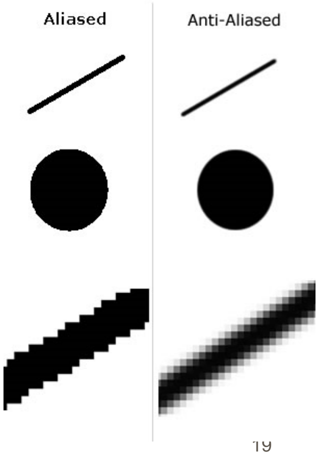

Aliasing


Anti-aliasing (multi-sampling)


Anti-aliasing (super-sampling)

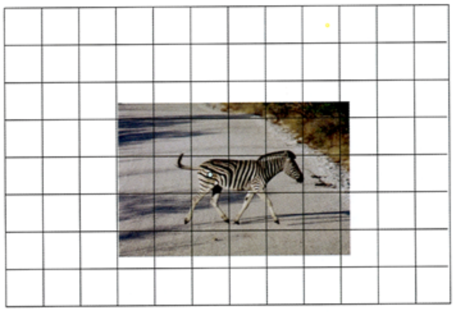
Aliasing

- Scene made up of black and white triangles: jaggies at boundaries
  - Jaggies will crawl during motion
- If triangles are small enough then we get random values or weird patterns.



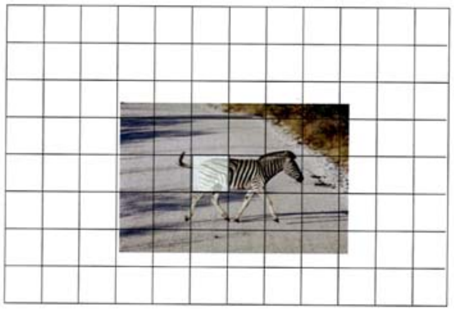
Aliasing

- The heart of the problem: too much information in one pixel



Anti-aliasing

- Intuitively: the single sample is a bad value, we would be better off setting the pixel value using some kind of average value over some appropriate region.
- In the above examples, perhaps some gray value.

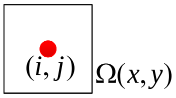


Anti-aliasing

- Mathematically this can be modeled using *Fourier analysis*.
  - Breaks up the data by “frequencies” and figures out what to do with the un-representable high frequencies.

Box filter

- We often choose the filters $F_{i,j}(x,y)$ to be something non-optimal, but that can more easily computed with.
- The simplest such choice is a *box filter*, where $F_{i,j}(x,y)$ is zero everywhere except over the 1-by-1 square center at $x = i, y = j$.

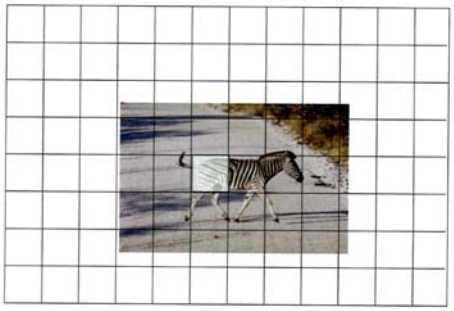


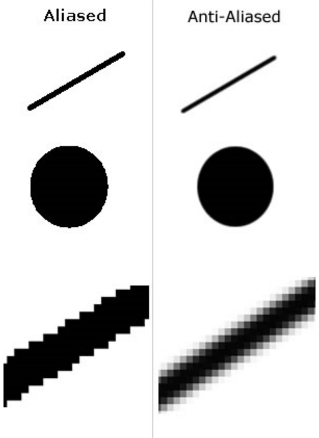
- Calling this square $\Omega_{i,j}$, we arrive at

$$\mathrm{I[i][j]} \leftarrow \iint_{\Omega_{i,j}} I(x,y)dxdy$$

Box filter

- In this case, the desired pixel value is simply the average of the continuous image over the pixel's square domain.

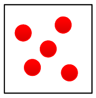




Over-sampling

- Even that integral is not really easy to compute
- Instead, it is approximated by some sum of the form:

$$\mathrm{I[i][j]} \leftarrow \frac{1}{n}\sum_{k=1}^{n} I(x_k, y_k)$$



  where *k* indexes some set of locations (x_k, y_k) called the sample locations.
- The renderer first produces a “high resolution” color and z-buffer “image”,
  - where we will use the term *sample* to refer to each of these high resolution pixels.