# CPSC 314
# Computer Graphics

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homogeneous transforms, rotation

## Announcements

- Quiz 1 graded will be available using cs handback, approx. Wednesday. Details on Wednesday.
- Assignment1 results will be available on Connect on or after Wednesday
- Assignment 2 available later this week.
- Today:
  - Essential math for graphics
    (read Textbook Chapter 2.5)

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# Tranforms, Rotations

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**Recap:** Homogeneous coordinates
in $n$ dimension, use $n+1$ numbers
to give coords to both points & vectors ($n = 3$, $= 4$)

Transformations 4 x 4 matrices

- Affine tranform

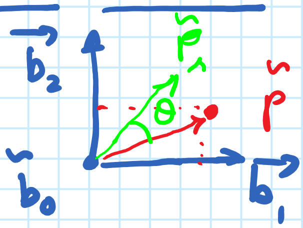
$$\bar{A} = \left[\begin{array}{c|c} L & t \\ \hline 0 & 1 \end{array}\right]$$

$L$ is any non-singular (invertible) 3x3 matrix

- Special cases
  - Identity $L = I_{3\times3}$
  - * Translation $\bar{A} = \left[\begin{array}{c|c} I_{3\times3} & t \\ \hline 0 & 1 \end{array}\right]$
  - * Scaling $\bar{A} = \left[\begin{array}{ccc|c} \alpha & & & \\ & \beta & & \\ & & \gamma & \\ \hline & & & 1 \end{array}\right]$
  - Reflections about different plane
  - * Rotations

## § Rotations in 2D

Use a reference frame $\bar{b} = (\vec{b}_1\ \vec{b}_2\ \tilde{b}_0)$, with $\vec{b}_1, \vec{b}_2$ unit vectors, orthogonal; $\tilde{b}_0$ point




$$\tilde{p} = \begin{pmatrix} p_1 \\ p_2 \\ 1 \end{pmatrix}$$

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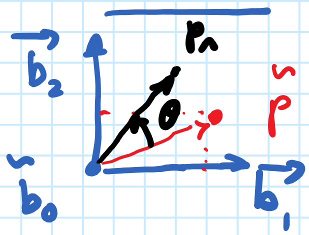
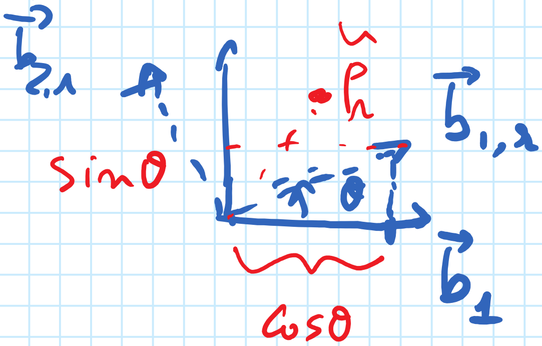



$$\bar{p} = \begin{pmatrix} p_1 \\ p_2 \\ 1 \end{pmatrix}$$

$$\bar{p}_r = ?$$

Let's first see what happens to the basis vectors




$$\vec{b}_{1,r} \rightsquigarrow \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix}$$

$$\bar{b}_{2,r} = \begin{pmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{pmatrix}$$

$$\bar{p}_r = \left[\begin{array}{cc|c} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ \hline 0 & 0 & 1 \end{array}\right] \begin{bmatrix} p_1 \\ p_2 \\ 1 \end{bmatrix}$$

$$\tilde{p}_r = \underline{\bar{b}}\,\bar{p}_r = \left( \vec{b}_1 \ \ \vec{b}_2 \ \ \tilde{b}_0 \right) \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} p_1 \\ p_2 \\ 1 \end{pmatrix}$$

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$$\hat{p}_r = \left( \vec{b}_{1,r} \quad \vec{b}_{2,r} \quad \hat{b}_0 \right) \begin{pmatrix} p_1 \\ p_2 \\ 1 \end{pmatrix}$$

So this the correct point obtained by rotatin the whole space by $\theta$ about the origin

$$\bar{p}_r = \underbrace{\left[\begin{array}{cc|c} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ \hline 0 & 0 & 1 \end{array}\right]}_{\bar{R}} \begin{pmatrix} p_1 \\ p_2 \\ 1 \end{pmatrix}$$

$\bar{R}$ homogeneous coordinates of a rotation in 2D

# Rotation in 3D

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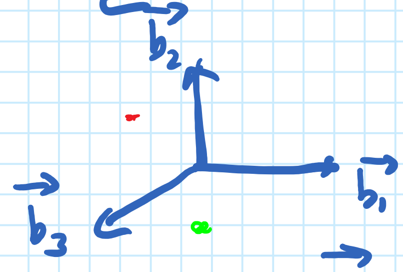
$$\bar{\bar{R}} = \left[\begin{array}{c|c} & \\ \hline & 1 \end{array}\right]$$

☆ Depends on axis of rotation

The example we just did was actually rotation about the "3" axis

[ Aside: $\vec{b}_x \equiv \vec{b}_1$, $\vec{b}_z \equiv \vec{b}_3$ etc.




Rotation about $\vec{b}_3$

$$Rot(z,\theta) = \left[\begin{array}{ccc|c} \cos\theta & -\sin\theta & & \\ \sin\theta & \cos\theta & & \\ & & 1 & \\ \hline & & & 1 \end{array}\right]$$

similarly

$$Rot(x,\theta) = \left[\begin{array}{ccc|c} 1 & \cdot & \cdot & \\ \cdot & \cos\theta & -\sin\theta & \\ \cdot & \sin\theta & \cos\theta & \\ \hline & & & 1 \end{array}\right]$$

$$Rot(y,\theta) = \left[\begin{array}{ccc|c} \cos\theta & \cdot & \sin\theta & \\ \cdot & 1 & \cdot & \\ -\sin\theta & \cdot & \cos\theta & \\ \hline & & & 1 \end{array}\right]$$

Note

☆ Any 3D rotation can be represented by a sequence of 3 rotations about axes