# CPSC 314
# Computer Graphics

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Two views of rotations.

Frame Transformations

## Announcements

- Quiz 1 and Assignment1 handback slightly delayed. Will be done by the end of the week
- Assignment 2 available later this week.
- Homework for today:
  - Read extbook Chapter 4, 5

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# Rotations, again

September 27, 2017 9:59 AM

$\underline{\underline{R}}$ — what is it about the matrix that makes it a rotation?

Each column and row has length 1 → 2-norm
The columns and rows are orthogonal

Such a matrix is called "orthogonal" matrix

Terminology: "orthonormal" refers to basis $\vec{b}_1, \vec{b}_2, \vec{b}_3$
"orthogonal" refers to a matrix.

Orthogonal matrices are nice!!

$$\underline{\underline{R}}^T \underline{\underline{R}} = \underline{\underline{I}}$$

 $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

So $\underline{\underline{R}}^{-1} = \underline{\underline{R}}^T$

Reflections are also orthogonal, but not Rotations

Rotations preserve "handedness"

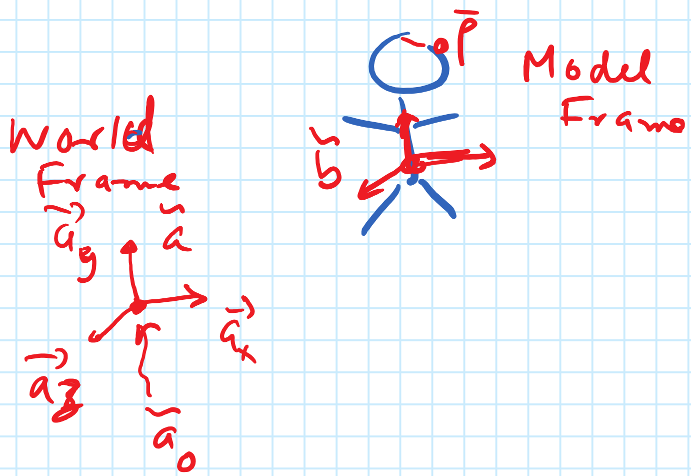
$\det(\underline{\underline{R}}) = +1$ Rotation ] only possible Orthogonal

$$\det(\underline{\bar{R}}) = +\underline{1} \quad \text{Rotation}$$
$$= -1 \quad \text{Reflection}$$

only possible Orthogonal Matrices

# Frame Transformations

September 27, 2017 10:28 AM




Two kinds of tasks

1. What are the coordinates of $\bar{p}$ in frame $\breve{a}$?
2. How do we move the model to another place?

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How to define a new frame $\underline{\breve{a}}$, relative to a known frame $\underline{\breve{b}}$?

The key is to realise that the physical point $\breve{p}$ doesn't change

$$\breve{p} = \underbrace{\underline{\breve{b}}\,\bar{p}}_{\text{given}} = \underline{\breve{a}}\,\underbrace{\bar{p}'}_{\text{find this}}$$

$$(\vec{a}_x \;\; \vec{a}_y \;\; \vec{a}_z \;\; \breve{a}_o)$$

$$\downarrow$$

$$(\underline{\breve{b}}\,\bar{a}_x \;\; \underline{\breve{b}}\,\bar{a}_y \;\; \underline{\breve{b}}\,\bar{a}_z \;\; \underline{\breve{b}}\,\bar{a}_o)$$

$$\downarrow$$

$$\underline{\breve{b}} \left( \bar{a}_x \,\middle|\, \bar{a}_y \,\middle|\, \bar{a}_z \,\middle|\, \bar{a}_o \right)$$

$$- \underbrace{\begin{pmatrix} a_x & a_y & a_z & a_0 \end{pmatrix}}_{\text{define as } \underline{\overline{A}}}$$

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$$\star \quad \breve{p} = \underline{\breve{b}}\,\bar{p} = \underline{\breve{a}}\,\bar{p}' = \underline{\breve{b}}\,\underline{\bar{A}}\,\bar{p}'$$

given find

$$\bar{p} = \underline{\bar{A}}\,\bar{p}'$$

$$\boxed{\bar{p}' = (\underline{\bar{A}})^{-1}\,\bar{p}}$$

Two views of what we just did, i.e.,

$$\underline{\breve{b}}\,\bar{p} = \underbrace{\underline{\breve{b}}\,\underline{\bar{A}}}\,\bar{p}'$$

1. Defined a new frame $\underline{\breve{a}}$
2. We moved $\bar{p}'$ by $\underline{\bar{A}}$ to get $\bar{p}$