

# Resampling

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Textbook Chapter 18

Several slides courtesy of M. Kim

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## Today

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- Announcements
  - Quiz 3 discussion
  - A4 grading
- Resampling

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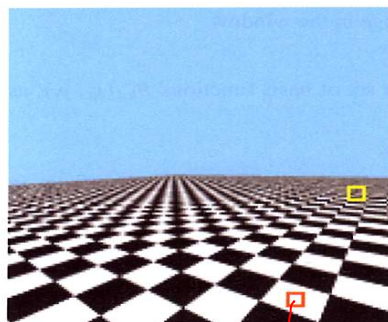
Chapter 18

## RESAMPLING

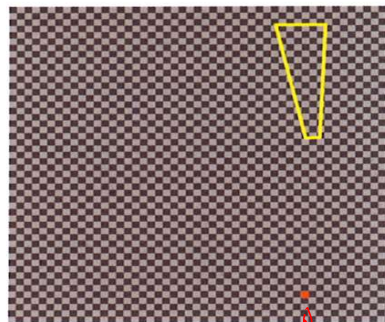
(RECONSTRUCTION+SAMPLING,  
DISCRETE→CONTINUOUS→DISCRETE)

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## Resampling



scan  
pixel



asmos packing  
footprint in texture

## Resampling

- Let's revisit texture mapping
- We start with a discrete image and end with a discrete image.
- The mapping technically involves both a reconstruction and sampling stage.
- In this context, we will explain the technique of mip mapping used for anti-aliased texture mapping.

*multum in parvo*

*Lance Williams*

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## (Textbook description) Resampling equation

- Suppose we start with a texture image (discrete)  $T[k][l]$  and apply some 2D warp to this image to obtain an output image  $I[i][j]$ .
- Reconstruct a continuous texture  $T(x_t, y_t)$  using a set of basis functions  $B_{k,l}(x_t, y_t)$ .
- Apply the geometric warp (at the view point) to the continuous image.
- Integrate against a set of filters  $F_{k,l}(x_w, y_w)$  (a box filter) to obtain the discrete output image.

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## (Textbook description) Resampling equation

- Let the geometric transform be described by a mapping  $M(x_w, y_w)$  which maps from continuous window to texture coordinates.

- We obtain:

$$I[i][j] \leftarrow \iint_{\Omega} F_{i,j}(x_w, y_w) \left( \sum_{k,l} B_{k,l}[M(x_w, y_w)] T[k][l] \right) dx_w dy_w$$

$$= \sum_{k,l} T[k][l] \left( \iint_{\Omega} F_{i,j}(x_w, y_w) (B_{k,l}[M(x_w, y_w)]) dx_w dy_w \right)$$

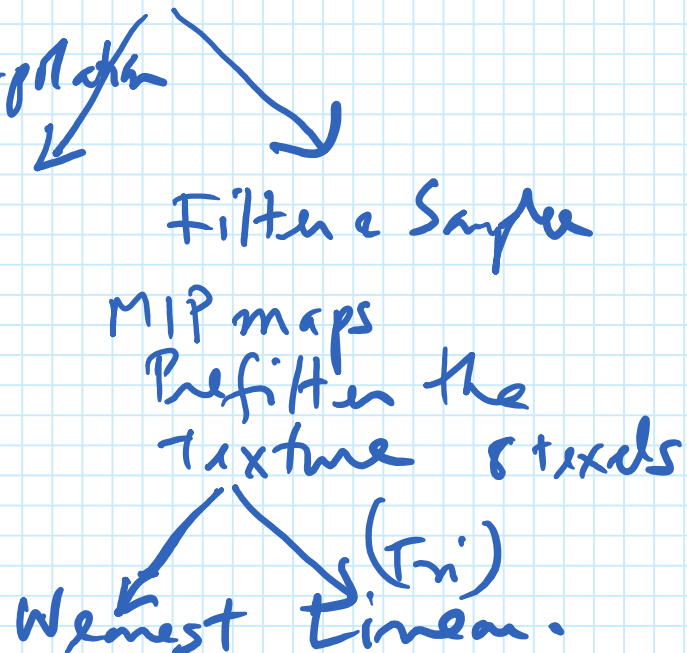
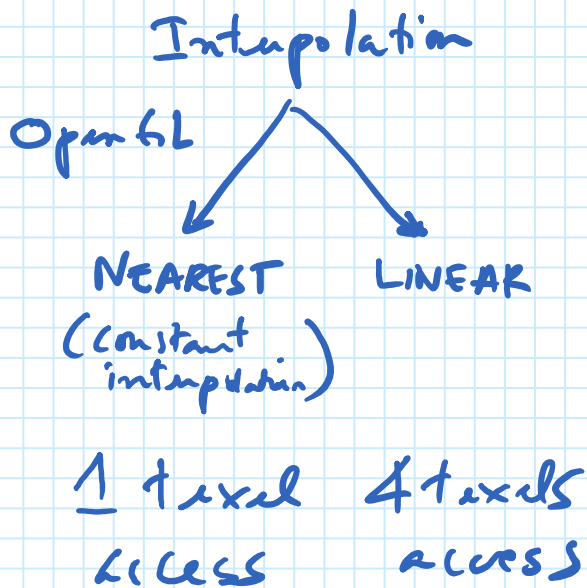
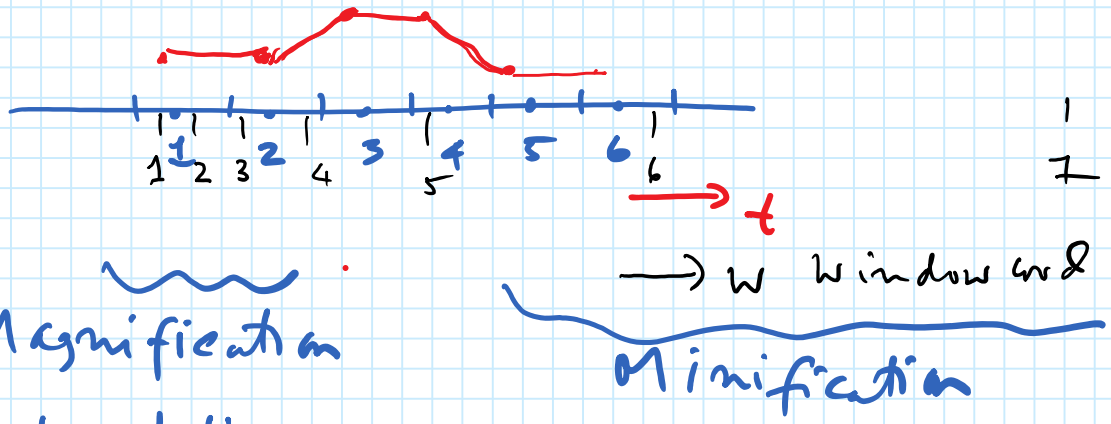
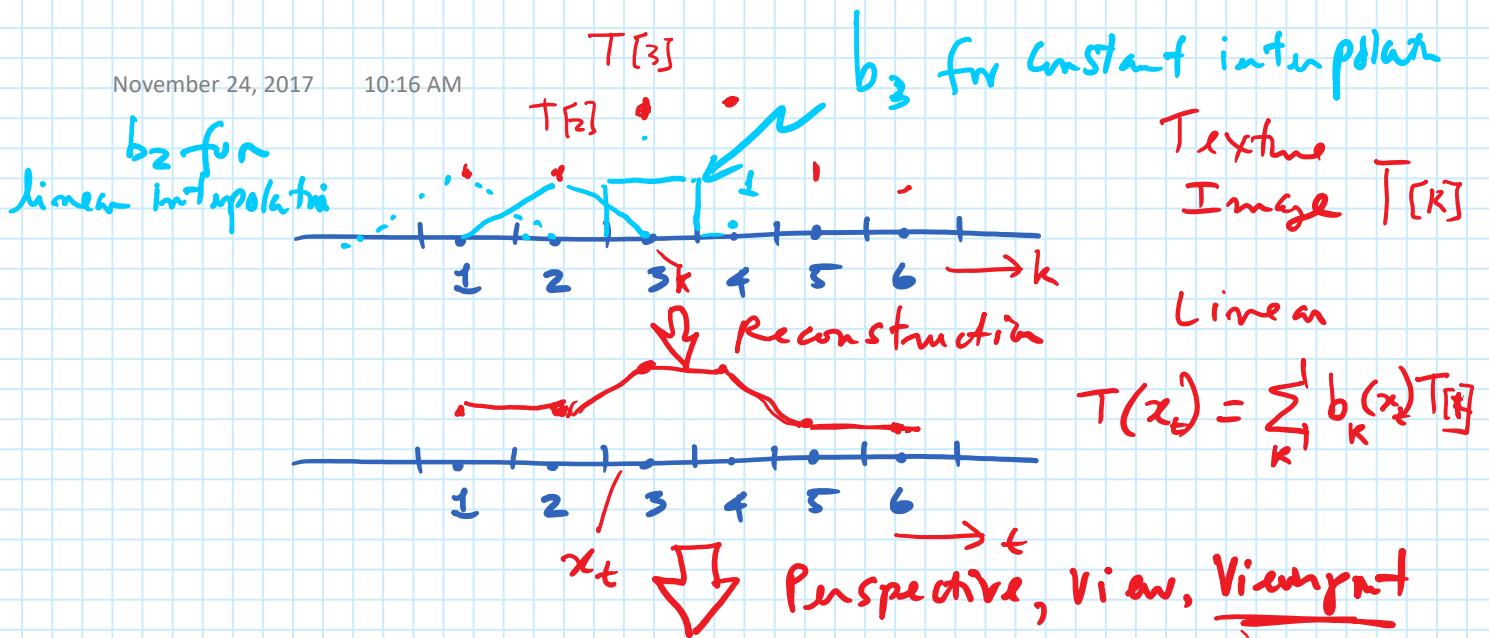
(we could obtain an output pixel as a linear combination of the input texture pixels.)

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## Key Intuition in 1D

- Switch to tablet

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## Magnification

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- We tell OpenGL to do this using the call `glTexParameteri(GL_TEXTURE_2D, GL_TEXTURE_MAG_FILTER, GL_LINEAR)`.
- In Three.js set `Texture.magFilter` to `THREE.LinearFilter` (default)
- For a single texture lookup in a fragment shader, the hardware needs to fetch 4 texture pixels and blend them appropriately.

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## Minification

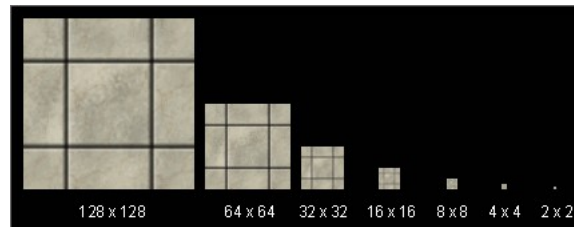
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- In the case that a texture is getting shrunk down, then, to avoid aliasing, the filter component should not be ignored.
- Unfortunately, there may be numerous texture pixels under the footprint of  $M(\Omega_{i,j})$ , and we may not be able to do our texture lookup in constant time.

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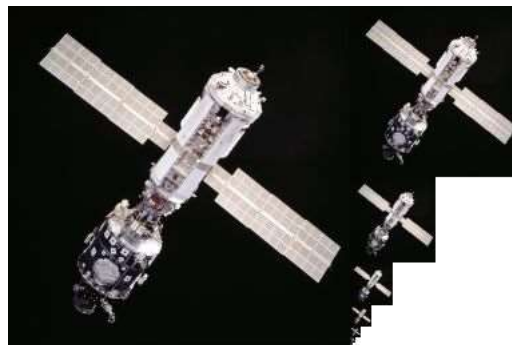
## Mip mapping

- In mip mapping, one starts with an original texture  $T^0$  and then creates a series of lower and lower resolution (blurrier) texture  $T^i$ . *for anti-aliasing?*
- Each successive texture is twice as blurry. And because they have successively less detail, they can be represented with  $\frac{1}{2}$  the number of pixels in both the horizontal and vertical directions.



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## Mipmap example

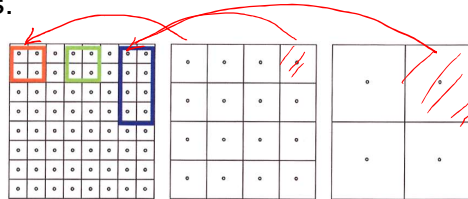


Source: wikipedia

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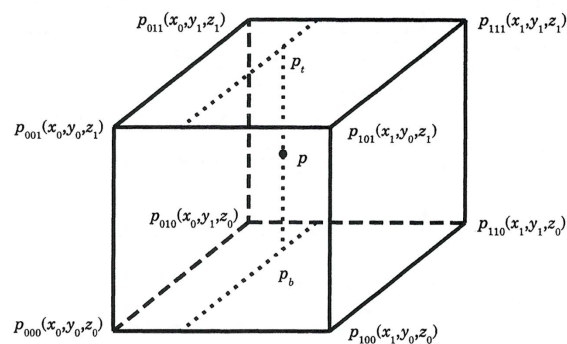
## Mip mapping

- In OpenGL/WebGL Mip mapping with trilinear interpolation is specified with the call `glTexParameteri(GL_TEXTURE_2D, GL_TEXTURE_MIN_FILTER, GL_LINEAR_MIPMAP_LINEAR)`
- In Three.js set `Texture.minFilter` to `THREE.LinearMipMapLinearFilter`
- Trilinear interpolation requires OpenGL to fetch 8 texture pixels and blend them appropriately for every requested texture access.



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## Trilinear interpolation



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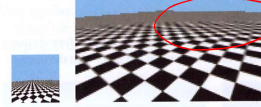
## Properties

- It is easy to see that mip mapping works reasonably well, but has limitations that can be addressed by more advanced methods.

No mip mapping



Mip mapping

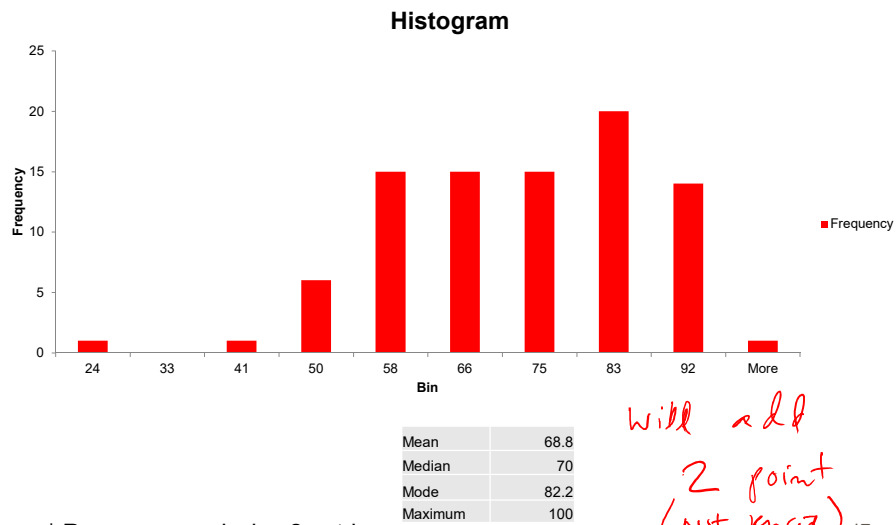


Anisotropic  
mip mapping



- Next class: Course review

## Quiz 3 performance\*



## Quiz 3 solutions

- Q1: 9,16,10,5,2,18,15
- Q2: T,F,T,T

## Quiz 3 solutions

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- Q3 (these example answers. Equivalent statements are acceptable.
  - a. key point is that the output is still in clip coordinates, and you have to do the “perspective divide” yourself
  - b. (from Lecture)

$$d_{45} s \mid (1-t)^3 \quad 3(1-t)^2 t \quad 3(1-t)t^2 \quad t^3$$

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- Q3 continued
  - c. from L27, the viewport matrix:

$$\begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix} = \begin{bmatrix} W/2 & 0 & 0 & (W-1)/2 \\ 0 & H/2 & 0 & (H-1)/2 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_n \\ y_n \\ z_n \\ 1 \end{bmatrix}$$

substitute W=512 H=256 (Ok if you exchanged W and H)

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■ Q3 continued

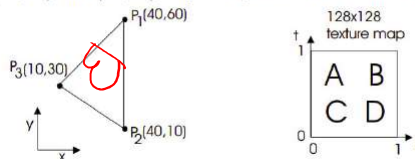
- d. pixel A:  $3/8 = 3/4 * 0.5$ , B: 0  
many forgot to multiply by intensity (0.5)  
B = 1 acceptable if assumption of background stated
- e. Key point is that the resolution of the depth buffer (z buffer) is limited, and what you are seeing is the aliasing in sampling the depth coarsely.

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■ Q4

triangle. The  $(s, t)$  texture coordinates of  $P_1$ ,  $P_2$ , and  $P_3$  are  $(0.5, 0.5)$ ,  $(1.0, 0.5)$ , and  $(0.8, 1.0)$ , respectively. Draw the textured triangle.



most people got this right.  
-1 for not flipping B

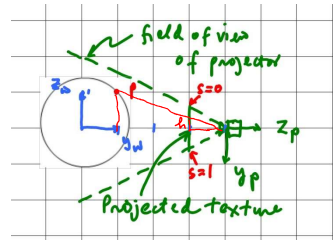
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■ Q6

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

Not Viewport

- a.  $[0 \ -1 \ 0; 1 \ 0 \ -4; 0 \ 0 \ 1]$  (for 2D view matrix)  
many did not have a rotation part.  
Some didn't have the position oriented properly
- B. answer =  $1/6$ . Many drew figure but didn't have right logic.



$$\frac{h}{1} = \frac{1}{3}$$

Simpler explanation than in class: Observe that the edge of the slide marked "s=0" is at height 0.5. So the distance from there the intersection of the ray is  $s = 1/2 - 1/3 = 1/6$

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